B.Sc. Part-I (Physics)

Date- 11-07-2020

Paper-I

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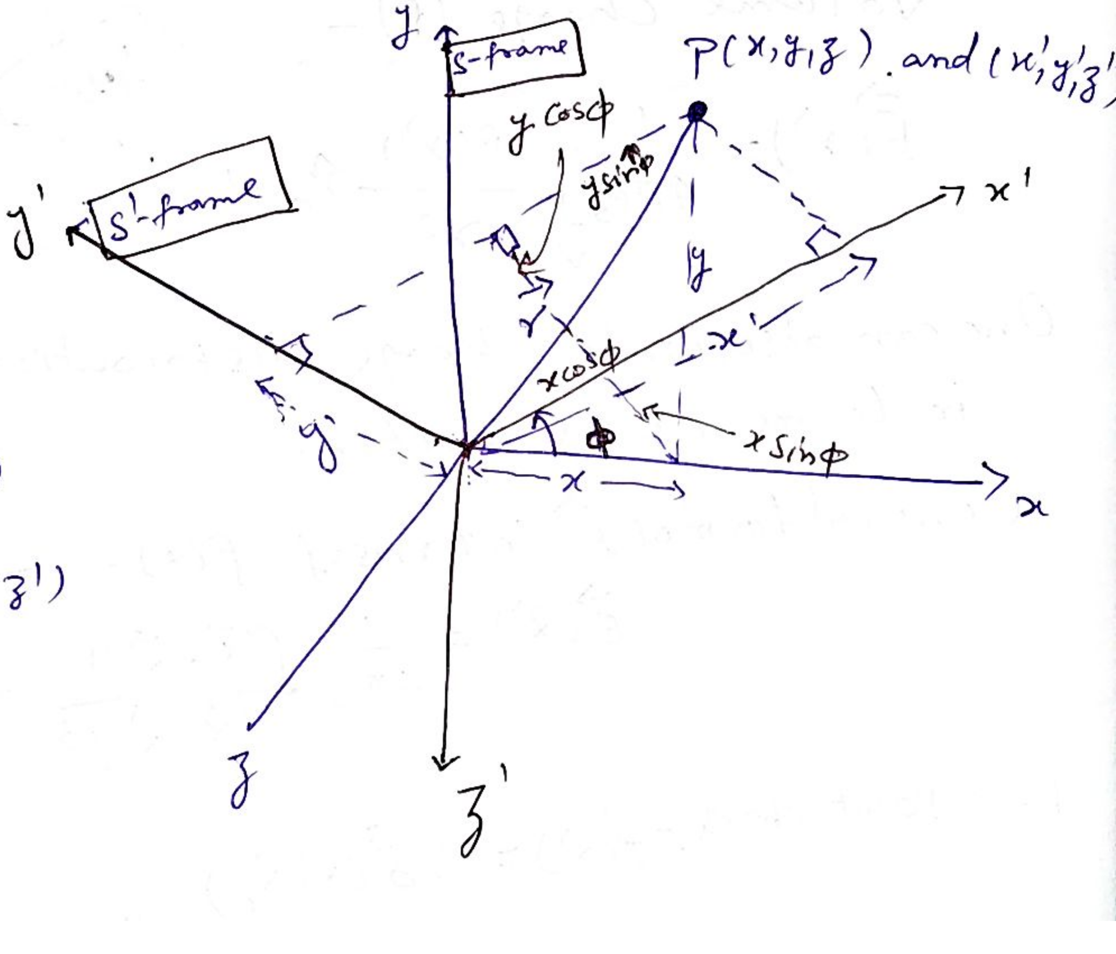
In earlier lectures & enotes, we have discussed inertial and non-inertial frames. Here we discuss transformation equations for a frame of reference inclined to or rotating to an inertial frame.

i) Transformation equations for a frame of reference inclined to an inertial frame.



Inertial frame

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S-frame (x, y, z)

S'-frame (x', y', z')

Let S'-frame is inclined with any angle ϕ from the S-fram

For simplicity we have considered here 2D case. And generalize it to 3D or higher dimensions.

Let $\vec{r} = x\hat{i} + y\hat{j}$

From Fig.

$$x' = x\cos\phi + y\sin\phi \quad \text{--- (1)}$$
$$y' = -x\sin\phi + y\cos\phi \quad \text{--- (2)}$$

If we take that z' has no inclination w.r.t S frame, $\Rightarrow z' = z$. — (3)

Now we try to generalize above relations.

We write
$$\boxed{x'_i = \sum_{j=1}^{2} a_{ij} \cdot x_j ,} \quad \text{--- (4)}$$

$a_{ij} \rightarrow$ direction cosine. $a_{ij} \equiv \cos(x'_i, x_j)$

In above case,

$$x' = a_{x'x} \cdot x + a_{x'y} \cdot y$$
$$y' = a_{y'x} \cdot x + a_{y'y} \cdot y$$

$$a_{x'x} = a_{11}(\text{let}) = \cos(x', x) = \cos\phi$$
$$a_{x'y} = a_{12}(\text{let}) = \cos(x', y) = \cos(\phi - \pi/2) = \sin\phi$$
$$a_{y'x} = a_{21}(\text{let}) = \cos(y', x) = \cos(\pi/2 + \phi) = -\sin\phi$$
$$a_{yy'} = a_{22}(\text{let}) = \cos(y, y') = \cos\phi .$$

For 3-D case above Eqn. (4)

$$x'_i = \sum_{j=1}^{3} a_{ij} \cdot x_j$$

$$\left.\begin{aligned} x_1' &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ x_2' &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ x_3' &= a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{aligned}\right\} \quad (5)$$

~~If we take~~ or in matrix form

or
$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (6)$$

~~If we take $x_1 \to x_1$, $y_1 \to y$, $z_1 \to z$~~

If we take $x_1 \to x$, $x_2 \to y$, $x_3 \to z$ & same for corresponding primed coordinate. we can write

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a_{x'x} & a_{x'y} & a_{x'z} \\ a_{y'x} & a_{y'y} & a_{y'z} \\ a_{z'x} & a_{z'y} & a_{z'z} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (7)$$

where $a_{xx} = \cos(x', x)$

$a_{y'x} = \cos(y', x)$, etc.

Note: $\cos(x', x)$ and other angles are constant, i.e, these are not changing with respect to time for the present case. They can change we will take that situation in next topic

Eq^n (5) or (6) or (7) are transformation equations from S-frame to S$'$-frame.

Now we differentiate w.r.t to time (t).

Let us consider x'-transformation equation of Eq. (7)

$$x' = a_{x'x}\, x + a_{x'y}\, y + a_{x'z}\, z$$

$$\frac{d^2x'}{dt^2} = a_{x'x}\frac{d^2x}{dt^2} + a_{x'y}\frac{d^2y}{dt^2} + a_{x'z}\frac{d^2z}{dt^2},$$

Similarly for y' and z'.

Since we have taken S-frame to be inertial.

Therefore, $\frac{d^2x}{dt^2} = 0$, $\frac{d^2y}{dt^2} = 0$ $\frac{d^2z}{dt^2} = 0$

$\Rightarrow$ $\frac{d^2x'}{dt^2} = 0$, and similarly $\frac{d^2y'}{dt^2} = 0$, $\frac{d^2z'}{dt^2} = 0$.

This implies that S$'$-frame is also an inertial frame of reference. And Eq^n (7) is Galilean transformation Eq.

For the inverse Galilean transformation Eq. of (7) we can write

$$\cancel{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{xx'} & a_{y'x} & a_{z'x} \\ a & & \\ & & \end{bmatrix}}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{xx'} & a_{xy'} & a_{xz'} \\ a_{yx'} & a_{yy'} & a_{yz'} \\ a_{zx'} & a_{zy'} & a_{zz'} \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad \text{--- (8)}$$